Thus the following theorem is proved.

Theorem. The arbitrary force

$$Q(q, q^{\cdot}) = Q^*(q) + Q^{**}(q^{\cdot})$$

that is continuous with its first order derivatives can be resolved into potential, nonconservative position, gyroscopic, and dissipative forces.

The definition (8) of nonconservative position forces and the definition (10) of gyroscopic forces imply that the first must depend on coordinates $q_k$ of the system, while the second depend on velocities $q_k^{\cdot}$. However, the general definition (8) of the nonconservative position forces does not exclude the possibility of these forces depending also on velocities $q_k^{\cdot}$ and time $t$. The gyroscopic and dissipative forces may, also, depend not only on velocities $q_k^{\cdot}$ but on coordinates $q_k$ and time $t$, as well. Certain theorems that determine the stability properties of motion of a system on the basis of force structure which satisfy these general defintions are given in [4].

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# ON CERTAIN INTEGRAL RELATIONSHIPS IN THE KINETIC THEORY OF GASES

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A method is proposed for the determination of certain moments of the Boltzmann collision integral, which appear in boundary problem solutions in the kinetic theory of gases, by expansion in the velocity half-space without actually calculating these [1], which makes it possible to establish definite relationships (including those derived earlier [2, 3]) between the moments.

The solution of boundary value problems of the kinetic theory of gases by the method of expansion in the velocity half-plane necessitates the determination of certain integrals that are moments of the Boltzmann collision integral, which many authors rightly consider to be the most laborious part of solving problems by this method. The step-by-step method of direct calculation of these moments was developed in [1]. It makes it possible, in principle, to solve a wide class of boundary value problems of the kinetic theory. In practice the application of that method [1] necessitates, however, very laborious calculations and does not provide means for checking the obtained results (errors appear in [1] in the determination of some of the moments), and, what is more important, the use of exact values of moments in the approximate theory leads in a number of cases to effects that must be attributed to excessive accuracy.

It was shown in [2, 3] how to establish certain relationships between moments of the collision integral on physical considerations and, thus, simplifying the process of solving the problem. A number of such relationships is established in the present work by mathematical means.

Let us consider the Boltzmann equation in the form proposed by Chapman-Enskog [4]

$$f^{(0)}\left[\left(c^2-\frac{5}{2}\right)c_i\frac{\partial T}{\partial r_i}+2\left(c_ic_j-\frac{1}{3}\delta_{ij}c^2\right)\frac{\partial c_{0i}}{\partial r_j}\right]=n^2I(\varphi)$$

Its solution is sought in the form

$$\varphi=\frac{1}{n}\left[A_i\frac{\partial \ln T}{\partial r_i}+B_{ij}\frac{\partial c_{0i}}{\partial r_j}\right] \qquad (1)$$

The method of successive approximations in the linear theory with respect to derivatives $\partial T/\partial r_i$ and $\partial c_{0i}/\partial r_j$ consists of taking into account of one, two, etc. terms of expansions of coefficients $A_i$ and $B_{ij}$ in Sonin-Laguerre polynomials. In particular, solution (1) in the approximation of a single polynomial (for any model of molecule interaction) is of the form

$$\varphi=-\frac{1}{n}\left[a_1\left(\frac{5}{2}-c^2\right)c_i\frac{\partial T}{\partial r_i}+b_1\left(c_ic_j-\frac{1}{3}\delta_{ij}c^2\right)\frac{\partial c_{0i}}{\partial r_j}\right] \qquad (2)$$

where coefficients $a_1$ and $b_1$ are related by formula

$$-4a_1=3b_1 \qquad (3)$$

In practice these are specified by the model of molecular interaction, which makes it possible to determine parameters $a_1$ and $b_1$. Thus for hard balls $b_1={}^5/_4\sqrt{\pi}\lambda$, where $\lambda$ is the molecule mean free path and for Maxwellian molecules

$$b_1={}^4/_3\,(m/2k)^{1/2}[\pi A_2(5)]^{-1},\quad A_2(5)=0.436$$

Let us consider the two-dimensional isothermic shear flow of gas along the axis $z$ ($\partial T/\partial r_i=0$). With allowance for (2) the Boltzmann equation has the form

$$2c_xc_z=b_1I(c_xc_z)$$

Let us consider moments $c_x$ sign $c_x$, $c_xc_z$, $c_zc_x^2$ sign $c_x$, $c_zc^2$ sign $c_x$ of this equation. As usual the moment $A(c)$ of function $\Phi(c)$ is taken to be the integral

$$\int A(c)\varphi(c)\exp(-c^2)\,d^3c$$

with respect to all velocities. Then in the right-hand part we obtain moments of the collision integral

$$I_2=[c_xc_z,\ c_z\operatorname{sign}c_x],\quad I_6=[c_xc_z,\ c_x^2c_z\operatorname{sign}c_x]$$
$$I_3=[c_xc_z,\ c_xc_z],\quad I_{15}=[c_xc_z,\ c_zc^2\operatorname{sign}c_x]$$

which is the aim of this paper.

Integrals in the left-hand part are elementary computed. As the result we have

$$I_2=-\pi/b_1,\quad I_6=-\pi/b_1,\quad I_3=-{}^1/_2\pi\sqrt{\pi}/b_1,\quad I_{15}=-3\pi/b_1 \qquad (4)$$

If there is only a temperature gradient ($\partial T/\partial r_i=\partial T/\partial z$) in the gas, then

$$({}^5/_2-c^2)\,c_z=a_1I\,[({}^5/_2-c^2)\,c_z]$$

Let us consider moments $c_x c_z \operatorname{sign} c_x$, $c_z\,(^5/_2 - c^2)$. A simple computation yields

$$I_{11} = \left[\left(\frac{5}{2} - c^2\right) c_z,\ c_x c_z \operatorname{sign} c_x\right] = -\frac{\pi}{4a_1}$$
$$I_{13} = \left[\left(\frac{5}{2} - c^2\right) c_z,\ \left(\frac{5}{2} - c^2\right) c_z\right] = \frac{5\pi\sqrt{\pi}}{4a_1} \tag{5}$$

The comparison of formulas (4) and (5) with allowance for (3) shows that the following relationships:

$$I_2 = I_6 = \frac{2}{\sqrt{\pi}} I_3, \quad I_{16} = 3I_2, \quad I_{11} = -\frac{1}{5\sqrt{\pi}} I_{13} = -\frac{1}{3} I_2 \tag{6}$$

between the moments of collision integrals are satisfied (independently of the selected model of molecular interaction).

In practical applications we have also to deal with moment $I_{12} = [c_x c_z,\ (^5/_2 - c^2)\, c_z \operatorname{sign} c_x]$

It will be readily seen that

$$I_{12} = -{}^1/_2 I_2 \tag{7}$$

Equalities (6) and (7) are corollaries of the single-polynomial approximation of expansion (1) whose accuracy depends on the model of molecular interaction (for Maxwellian molecules it is exact). It should be noted that the method of half-space expansions proposed in [5] and later developed in the kinetic theory of gases in [6] was never extended beyond a single-polynomial approximation (for higher approximations this method becomes much more complex; the author is not aware of any publications using such approximations). Hence a nonobservance of equalities (6) and (7) when solving boundary value problems by the method (6) must be considered as leading to excess (illusory) accuracy (see in the connection with this [2, 3] and also [7, 8]).

Thus the values of certain moments of Boltzmann collision integrals (in linearized form) can be determined without a direct and fairly laborious computation. Such values are exact for Maxwellian molecules. For other models their accuracy is limited by the single-polynomial of the expansion by the Chapman-Enskog method. It should be stressed that more exact values of moments used in some publications may in many cases lead (and do so) to the prediction of effects that do not exist in reality and are simply the result of exceeding the exactitude of the theory. The method described in this paper satisfies the obligatory physical requirement [2] that the correct passage to limit in the volume of gas away from walls must be ensured in solutions of boundary value problems of the kinetic theory.

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# A MATHEMATICAL MODEL OF AIRCRAFT FOR THE INVESTIGATION OF NONSTATIONARY AERODYNAMIC CHARACTERISTICS

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Basic proportions are defined for setting a computation scheme for the determination of aerodynamic characteristics of an aircraft on a computer. A schematic representation of an aircraft by a system of reference elements is described and substantiated. The general nonstationary linear problem of aerodynamics and hydrodynamics is reduced to a set of canonical $\varepsilon$ -problems that are solved independently. General properties of linear characteristics (loads, normal forces, and moments) are established. Integral relationships of the convolution kind which yield explicit formulas for these characteristics in terms of related transition functions and laws of kinematic parameter variations with time $\varepsilon_j(\tau)$ are derived. A number of general theorems, including the generalization of the invertibility theorem, are proved. Exact formulas are established for transition functions at the initial instant of time.

**1. The general nonstationary linear problem.** Let us consider the unsteady motion and the deformations of an aircraft in a perturbed continuous medium. We attach to the aircraft a conventional Cartesian system of coordinates whose $Ox$-axis is directed forward along the aircraft axis and the $Oz$-axis directed to the right over the wing span (Fig. 1). Let $U_0$ be the mean velocity of the (coordinate) origin 0, and $W$, $W^*$, $W_\Delta$ and $W_\delta$ be the velocity vectors of perturbations induced by the aircraft, transfer, gusts of the medium, and aircraft surface deformations, respectively. We denote the vector of absolute angular velocity by $\Omega$ and time by $t$ ($t = 0$ is the instant of unstable processes onset), and introduce the dimensionless quantities

$$\xi = \frac{x}{b}, \quad \eta = \frac{y}{b}, \quad \zeta = \frac{z}{b}, \quad \tau = \frac{u_0 t}{b}$$

$$\omega_{x,y,z}(\tau) = \frac{b\Omega_{x,y,z}}{U_0}, \qquad w_{x,y,z\Delta}(\xi, \eta, \zeta, \tau) = \frac{W_{x,y,z\Delta}}{U_0}$$

where $b$ is a characteristic linear dimension.

Let $uU_0$ be the varying velocity component of origin $O$, and $\alpha$ and $\beta$ the angles of attack and of the side slip, respectively. For projections of the transfer velocity at any point of the aircraft surface we then have